١٧ $\int \frac{\text{لو}_{\text{ط}} \text{جا}\, س}{\text{جا}^{2} س}\, \text{د}س$

$ص = \text{لو}_{\text{ط}} \text{جا}\, س$

$\text{د}ص = \frac{\text{جتا}\, س}{\text{جا}\, س \times \text{لو}_{\text{ط}}}\, \text{د}س$

$\text{د}س = \frac{\text{جا}\, س \times \text{لو}_{\text{ط}}}{\text{جتا}\, س}\, \text{د}ص$

$\text{د}س = \frac{\text{لو}_{\text{ط}}}{\text{جا}\, س \text{ ظا}\, س}\, \text{د}ص$

$\text{د}س = \text{جا}\, س \text{ ظا}\, س \text{ د}ص$

$\int \frac{ص}{\text{جا}\, س \text{ ظا}\, س} \times \text{جا}\, س \text{ ظا}\, س \text{ د}ص$

$= \int ص^{2} \text{ د}ص = \frac{1}{3} ص^{3} + ط$

$= \frac{1}{3} (\text{لو}_{\text{ط}} \text{جا}\, س)^{3} + ط$

$\int \frac{\text{د}س}{\sqrt{س^{2} + 1}}$

$\int \frac{\text{د}س}{\sqrt{س^{2} (1 + \frac{1}{س^{2}})}}$

$\int \frac{\text{د}س}{س \sqrt{1 + \frac{1}{س^{2}}}}$

$\int \frac{\text{د}س}{س^{2} \sqrt{1 + \frac{1}{س^{2}}}}$

$\int \frac{1}{س^{2}} (1 + \frac{1}{س^{2}})^{-\frac{1}{2}}\, \text{د}س$

$ص = 1 + \frac{1}{س^{2}}$

$\text{د}س = \frac{\text{د}ص}{-\frac{2}{س^{3}}} = -\frac{س^{3}}{2}\, \text{د}ص$

$\int \frac{1}{س^{2}} ص^{-\frac{1}{2}} \times -\frac{س^{3}}{2}\, \text{د}ص$

$\int -\frac{1}{2} ص^{-\frac{3}{2}}\, \text{د}ص = -\frac{1}{2} \times -2 ص^{-\frac{1}{2}} + ط$

$= \frac{1}{ص} (1 + \frac{1}{س^{2}})^{-\frac{1}{2}} + ط$

١٨ $\int \frac{(س^{3} + 3)^{9}}{س^{11}}\, \text{د}س$

$ص = 3 + \frac{1}{س^{3}}$

$\text{د}س = \frac{\text{د}ص}{-\frac{3}{س^{4}}} = -\frac{س^{4}}{3}\, \text{د}ص$

$\int \frac{(س^{3} (1 + \frac{3}{س^{3}}))^{9}}{س^{11}}\, \text{د}ص$

$\int \frac{س^{27} (1 + \frac{3}{س^{3}})^{9}}{س^{12} \times س^{-1}}\, \text{د}ص$

$\int -\frac{1}{3} ص^{9}\, \text{د}ص = -\frac{1}{3} \times \frac{1}{10} ص^{10} + ط$

$= -\frac{1}{30} (3 + \frac{1}{س^{3}})^{10} + ط$

١١٥) $\int \tan x \sec^{-3} x \, dx$

$\int \tan x \times \frac{1}{\sec^{2} x} \, dx$

$\int \tan x \sec^{-2} x \, dx$

ص = $\sec^{-2} x$

$dx = \frac{dy}{\sec^{2} x}$

$\int$ ص $\sec^{2} x \times \frac{dx}{\sec^{2} x}$

$= \frac{\sec^{2} x}{2} + C = \frac{ص^{2}}{2} + C$

١١٨) $\int \frac{x^{2}\sec x + \sec x \tan x}{x^{2} + \sec x} \, dx$

$\int \frac{\sec x \times \frac{\tan x}{x}}{\frac{1}{x} \times \tan x} \, dx$

$\frac{\frac{x \sec x + \tan x}{x}}{\frac{1 + x\tan x}{x}}$

$\int \frac{x \sec x + \tan x}{1 + x \tan x} \, dx$

$= \ln |1 + x \tan x| + C$

١١٦) $\int x^{2} (2\sin x \cos x - \cos 2x) \, dx$

$\int x^{2} (2 \times \frac{1}{2}(\sin 2x + \sin 0x) - \cos 2x) \, dx$

$\int x^{2} (\cos 2x - \sin 2x + \sin 2x) \, dx$

$\int x^{2} \cos 2x \, dx$

ع	م
$\cos 2x$	x^{2}
$\frac{1}{2}\sin 2x$	$2x$
$\frac{-1}{4}\cos 2x$	2
$\frac{-1}{8}\sin 2x$	صفر

$\int x^{2} \cos 2x \, dx = \frac{1}{2} x^{2} \sin 2x - \frac{2}{4} x \cos 2x + 2 \times \frac{-1}{8} \sin 2x + C$

$= \frac{1}{2} x^{2} \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x + C$

(١٦) $\int \frac{\text{جتا}^2 س \, \text{جا} س}{\text{جتا}^3 س} \, دس$

$\int \frac{\frac{1}{2}(\text{جا}^2 س + \text{جا}^2 س)}{\text{جتا}^3 س} \, دس$

$\int \frac{\text{جا}^2 س}{\text{جتا}^3 س} \, دس + \int \frac{\text{جا}^2 س}{\text{جتا}^3 س} \, دس$

$\int \frac{\text{جا}^2 س \, \text{جتا}^2 س}{\text{جتا}^3 س} + \text{ظا}^3 س \, دس$

$\int \text{جا} س \, \text{ظا}^2 س \, دس$

$= -\frac{1}{2} \text{جتا}^2 س - \frac{1}{2} \ln|\text{جا}^2 س| + جـ$

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(٢٢) $\int \frac{\text{ظا} س}{(٤ - (\ln \text{جا} س)^2)} \, دس$

$ص = \ln \text{جا} س$

$دس = \frac{دص}{\frac{\text{جا} س}{\text{جتا} س}} = \frac{دص}{\text{ظا} س}$

$\int \frac{\text{ظا} س}{٤ - ص^2} \, \frac{دص}{\text{ظا} س}$

$\int \frac{1}{(٢ - ص)(٢ + ص)} \, دص$

$\int \frac{أ}{٢ - ص} + \frac{ب}{٢ + ص} \, دص$

$أ(٢ + ص) + ب(٢ - ص) = ١$

$أ = \frac{1}{4}$ ، $ب = \frac{1}{4}$

$\int \frac{\frac{1}{4}}{٢ - ص} + \frac{\frac{1}{4}}{٢ + ص} \, دص$

$= -\frac{1}{4} \ln|٢ - \ln \text{جا} س| + \frac{1}{4} \ln|٢ + \ln \text{جا} س| + جـ$

(٢٣) $\int \ln(س^2 - ٦س + ٩) \, دس$

$= س \ln(س^2 - ٦س + ٩) - \int س \times \frac{٢س - ٦}{س^2 - ٦س + ٩} \, دس$

$= س \ln(س^2 - ٦س + ٩) - \int \frac{٢س^2 - ٦س}{س^2 - ٦س + ٩} \, دس$

$\begin{array}{r} ٢ \\ س^2 - ٦س + ٩ \overline{)\ ٢س^2 - ٦س} \\ \underline{٢س^2 - ١٢س + ١٨} \\ ٦س - ١٨ \end{array}$

$= س \ln(س^2 - ٦س + ٩) - \int ٢ + \frac{٦س - ١٨}{س^2 - ٦س + ٩} \, دس$

$= س \ln(س^2 - ٦س + ٩) - \int ٢ + \frac{٣(٢س - ٦)}{س^2 - ٦س + ٩} \, دس$

$= س \ln(س^2 - ٦س + ٩) - ٢س - ٣ \ln(س^2 - ٦س + ٩) + جـ$

٤٩١ $\int_0^{\pi} \ln(\sin^2 x - 2\sin x)\, dx$

$u = \ln(\sin^2 x - 2\sin x)$ $\quad dv = dx$

$du = \frac{2\sin x\cos x - 2\cos x}{\sin^2 x - 2\sin x}\, dx$ $\quad v = x$

$\int_0^{\pi} \ln(\sin^2 x - 2\sin x)\, dx = [x\ln(\sin^2 x - 2\sin x)]_0^{\pi} - \int_0^{\pi} \frac{2\sin x\cos x - 2\cos x}{\sin^2 x - 2\sin x}\, dx$

$= [\pi\ln(\sin^2 \pi - 2\sin \pi) - 0\ln(\sin^2 0 - 2\sin 0)] - \int_0^{\pi} \frac{2\cos x(\sin x - 1)}{\sin x(\sin x - 2)}\, dx$

$= \pi\ln(0 - 0) - \int_0^{\pi} \frac{2\cos x}{\sin x}\, dx$

$= \pi\ln(0) - \int_0^{\pi} 2\cot x\, dx$

$= \pi\ln(0) - [2\ln|\sin x|]_0^{\pi}$

$= \pi\ln(0) - [2\ln(0) - 2\ln(0)]$

$= \pi\ln(0)$